Phase-locking of multicore fibre laser due to Talbot self-reproduction

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Abstract. The multicore fibre laser (MCFL), containing an array of single-mode microcores in a circle inside the pump core, is a promising compact laser source. The problem is to synchronize the radiation of microcores with different propagation constants at a given radiation frequency. The theory of phase-locking of an MCFL with an external mirror and an annular waveguide matched to the multicore fibre and having a length some fraction of the Talbot distance is developed. Collective mode selection appears to occur due to spatial filtering at fractional Talbot distances, while the radiation frequency self-adjusts within the spectral gain range to minimize losses. Parallel coupling between microcores is achieved in the limit of a small fill factor. The maximum number of microcores that can be coupled is found. Good agreement is achieved between the theory and previously published experimental results.

1. Introduction

Diode-pumped fibre lasers are widely used in commercial applications as powerful and user-friendly sources of high brightness radiation. The usual end-pumped fibre laser consists of a fibre containing an active medium and two mirrors butt-coupled to its ends (figure 1). One of the mirrors (M₁) is almost transparent to pump radiation and completely reflecting to the laser radiation. This mirror is used to couple the pump radiation into the fibre. The other mirror (M₂) is semi-transparent to the laser radiation and serves as the output mirror. Multicore fibre (MCF) has been suggested as a way of reducing the fibre length necessary to absorb the pump radiation and increasing the maximum laser output power. A version of such a fibre, proposed in [1], consists of N single-mode waveguiding microcores placed in a circle inside a multimode fibre (figure 1). The microcores are doped with Nd³⁺ and pumped by diode laser radiation propagating in multimode fibre.

The disadvantage of the MCF laser is the poor output beam quality due to the small microcore aperture operating in a regime of independent lasing in microcores. The output wave field distribution of an MCF phase-locked with zero phase difference between microcores has an output radiation intensity on axis, that is N times greater than in the case of independent lasing. It was shown in gas laser studies [2] that a promising approach for achieving phase-locked operation of a laser array is to use the Talbot effect [3, 4] in which self-reproduction of a transversely periodic wave field distribution occurs over a distance $Z_T = 2\lambda^2/\lambda$, where $\lambda$ is the laser wavelength and $\lambda$ is the wavelength of the pump radiation.
where $\lambda$ is radiation wavelength and $\Lambda$ is the transverse period of the wave field pattern. However, the extension of Talbot cavity theory to a circular array of emitters, such as the MCF, remains to be studied.

Generally, the term ‘phase locking’ assumes that there exist some fixed phase differences between lasers. However, the far-field pattern from the laser array is very sensitive to exact values of these phase differences. Maximum radiance on the axis is achieved for all phase differences equal to zero. If the established phase relations between lasers are different, additional measures are necessary to improve combined beam optical quality.

Diffraction coupling between waveguiding microcores is very weak, and cannot provide phase locking. Additional measures should be undertaken to strengthen optical coupling between microcores. The simplest way [2] is to arrange an external Talbot cavity, consisting of the MCF and a plane mirror. It was demonstrated [5] that near out-of-phase lasing can be achieved when the external mirror is placed at some fraction of the Talbot distance from the MCF. The out-of-phase field distribution with phase difference $\pi$ between neighbouring microcores can be transformed into an in-phase distribution by an additional phase plate. Analysis of this MCFL construction, made in [6], has shown that diffraction in the radial direction results in irreversible beams expansion, which destroys the azimuthal Talbot effect and introduces high losses. It was suggested in [7] that an annular waveguide (AW) be placed between the MCF and the external mirror to diminish losses. The AW (figure 1) is a multimode fibre containing a waveguiding ring, with index, radius, and ring width chosen to confine the radiation emitted by the microcores. As a result, a near out-of-phase output distribution was obtained in the experiment [7] with a good net efficiency.

Supermode selection in the MCF with different versions of external Talbot cavity was considered in [6, 7] under the assumption that all the microcores are identical. However the fibre fabrication process breaks the identity of the microcores. Optical misalignments of composite cavity elements also disturb this assumption. The spread in optical lengths is usually much larger than the radiation wavelength. Thus, it is necessary to understand the mechanism of phase-locked lasing observed experimentally in these conditions. Phase-locking of the MCF laser with an external plane mirror and an AW of length $Z_{T}/4$ is shown below to occur due to parallel coupling provided by the fractional Talbot effect in a

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**Figure 1. Schematic of the MCF fibre laser with annular waveguide and external mirror:**

MCF is the multicore fibre; AW is the annular waveguide; M1 is the mirror transparent at the pump radiation wavelength $\lambda_p$ and reflecting at the lasing wavelength $\lambda_L$; M2 is the mirror semi-transparent at the wavelength $\lambda_L$. 

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ring system. The MCF investigated in the experiment [7] contains \( N = 18 \) microcores placed in a circle of radius \( R_C = 131 \mu m \) in the pump core of radius 145 \( \mu m \). The thickness of the AW was three times greater than the microcore diameter 8\( \mu m \), and the circle containing the microcores does not match an AW medium line. This results in excitation of a few radial modes in the AW, which in turn disturbs the Talbot effect.

2. **Fractional Talbot effect**

A periodic field distribution is known [8] to have multiple images after propagation over \((p/q)Z_T\) where \( p, q \) are integer numbers and \( Z_T \) is the Talbot distance; this is the so-called fractional Talbot effect. This phenomenon, analogous to fractional revivals in Rydberg atoms [9], has been used to increase the laser array aperture fill factor [10] (see also review [11]), for image synthesis [12], array illuminators [13, 14], and in atomic matter waves optics [15]. For example, the Talbot array illuminator, used in optical interconnections and multiple imaging, provides a 1D or 2D array of light illuminations. It is based on the fact that an amplitude grating can generate pure phase distributions at fractional Talbot distances, and so, conversely, it is possible to use phase modulation for array illumination. For an amplitude grating with an opening ratio of \( 1/M \) (\( M \) is an integer), pure phase distributions have been found to occur at \( pZ_T/(2M) \), where \( p \) is prime to \( M \) (see [16] and references therein). A set of equations has been derived to calculate the field distributions at the distances \((p/M)Z_T\) [17].

The approach used below to calculate the field distribution at fractional Talbot distances is based on the Green function for the field propagation problem with periodic initial condition. Radiation propagation in empty space is described in the scalar paraxial approximation by a parabolic equation for the field amplitude \( u(x, z) \):

\[
2i k \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} = 0, \tag{1}
\]

where \( z \) is along the direction of propagation, \( k \) is the wavevector in space. If an infinite periodic array of delta-functions with the period \( \Lambda \): \( \sum_{m=-\infty}^{\infty} \delta(x - m\Lambda) \) is taken as an initial field distribution, then the field distribution after propagation a distance \( pZ_T/q \) can be expressed in the form:

\[
u_A \left( \frac{pZ_T}{q}, x \right) = \sqrt{\frac{q}{2ip\Lambda^2}} \exp \left( \frac{i\pi qx^2}{2p\Lambda^2} \right) \sum_{m=-\infty}^{+\infty} \exp \left( -i\pi qmx \left( \frac{p}{\Lambda} \right) + \frac{i\pi qm^2}{2p} \right). \tag{2}\]

The delta-function \( \delta(x) \) corresponds to a wave field concentrated at \( x = 0 \), and zero everywhere else. The solution to the Cauchy problem with a delta-function array as the initial distribution can be obtained by convolution of the initial field with the Green function. Replacing the summation index \( m \) with \( l = (m-s)/2p \), where \( s = 0, 1 \ldots (2p-1) \), and using the Poisson relationship: \( \sum_{l=-\infty}^{+\infty} \exp(-ilx) = 2\pi \times \sum_{l=-\infty}^{+\infty} \delta(x - 2\pi l) \), the field distribution at the distance \( z = pZ_T/q \) can be written as:

\[
v_A(pZ_T/q, x) = (2ipq)^{-1/2} \sum_{l=-\infty}^{+\infty} \sum_{s=0}^{2p-1} \exp \left( i\pi \frac{qs^2}{2p} - i\pi \frac{sl}{p} \right) \exp \left( \frac{i\pi l^2}{2pq} \right) \delta(x - l\Lambda/q). \tag{3}\]
It can be seen that the field distribution is composed of no more than $q$ delta-
functions within a unit cell with relative phase shifts that can easily be calculated for a
given distance. In a fundamental study of the Fresnel images of a grating made by
Berry and Klein [18], it was found that the phases of the Talbot images possess an
irreducible arithmetic complexity and, for gratings with sharp-edged slits, the light
intensity has a fractal structure. Besides, it was found, that both a finite number of
grating slits and non-paraxial effects result in blurring of Talbot images.

For the case $p=1$, the formula (3) is reduced to the Guigai’s equation [19]:

$$u_A(Z_T/q, x) = (2iq)^{-1/2} \sum_{l=-\infty}^{+\infty} \exp(i \pi l^2 / 2q) \left[ 1 + \exp(-i \pi l + i \pi q/2) \right] \delta(x - l \Lambda/q).$$  (4)

The formulae (3) and (4) can be used to calculate the resulting field distribution for
any initial field profile. Notice that all the replicas resulting from propagation over
the fractional Talbot distance have the same width as the initial one. If the images
also overlap, the resulting field has complicated phase and amplitude profiles.
An appropriate parameter controlling overlap of replicas is so-called fill-factor,
which is defined as a ratio of emitting area to the whole area. In these terms the
array fill-factor has to be small enough to observe image multiplication.

The MCF output radiation can be expressed as a sum of supermodes indexed
by $K$, so that the phase difference between the two consecutive microcores is equal
to $2\pi K/N$ for the $K$th supermode. The field distribution with uniform phase
profile corresponds to the in-phase mode ($K=0$). In practice, the out-of-phase
mode ($K=N/2$) with phase difference $\pi$ between the nearest emitters is usually the
most difficult to suppress [20]. The Green function for such a field profile
describes the field propagation with the initial distribution $\sum_{m=-\infty}^{+\infty} \exp(i \pi m) \delta(x - m \Lambda)$. The resulting field distribution for distance $p Z_T/q$ is expressed as:

$$u_A(p Z_T/q, x) = (2ipq)^{-1/2} \sum_{l=-\infty}^{+\infty} \left( \sum_{s=0}^{2b-1} \exp \left\{ i \pi s [1 - (l - qs/2)/p] \right\} \right) \times \exp \left( \frac{i \pi l^2}{2pq} \right) \delta(x - l \Lambda/q).$$  (5)

This formula is reduced, for the case $p=1$, to:

$$u_A(Z_T/q, x) = (2iq)^{-1/2} \sum_{l=-\infty}^{+\infty} \exp(i \pi l^2 / 2q) \left[ 1 - \exp(-i \pi l + i \pi q/2) \right] \delta(x - l \Lambda/q).$$  (6)

Thus the out-of-phase mode is reproduced at fractions of Talbot distances with
the same image multiplication as for in-phase mode. It is seen from equations (4)
and (6), that at distances $Z_T/(2j)$, where $j$ is an integer, both the in-phase and out-
of-phase modes produce only $j$ replicas, which are located at different positions
within a period for the two modes. For $q = 4j$, the in-phase mode multiple images
are shifted (relative to the origin) to $0$, $\Lambda/2j$, $\ldots$, $\Lambda(2j-1)/2j$, while the out-of-phase
mode images are shifted to $\Lambda/4j$, $3\Lambda/4j$, $\ldots$, $\Lambda(1-1/4j)$.

† Two definitions of Talbot length exist in literature differing two times from each other.
Talbot length in [18] is defined as $\Lambda^2/\lambda$. 

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3. Supermode selection by virtue of fractional Talbot effect

The modes of an MCF laser cavity can be found by the usual method of cavity round-trip iteration. As a start, a linear combination of the microcore modes with equal amplitudes and phases, each concentrated at a location of the microcore, can be taken. In a round-trip over the MCF the wave field has an amplitude and phase dependence on the microcore number. This dependence can be attributed to variable gain and loss in each microcore, and to different propagation constants. Neglecting the difference between optical modes of the microcores, but keeping optical phase and amplitude differences appearing after the round-trip, the emitted wave field can be expressed in the form:

$$u_{\text{out}}(r) = \sum_{m=0}^{N-1} a_m \exp(i\varphi_m)f(r - r_m),$$

where $a_m$, $\varphi_m$ are the amplitude and phase shift of wave field emitted by the $m$th microcore; correspondingly, $f(r)$ is a mode profile of the microcore with centre at location $r_m$, radius vector is 2D vector in the MCF output facet plane. All the microcores centres are assumed to be positioned with equal distance $\Delta$ between them in a circle with radius $R_C$ centred at the MCF axis.

The annular waveguide confines only the one radial mode, so the field profile in the radial direction is strictly assigned. In these conditions, the field propagation through the AW is equivalent to propagation through a laterally unbounded planar waveguide with an additional condition that the wave field is periodic in the lateral direction with period equal to the circle length $2\pi R_C$ [21]. A similar situation takes place in so-called multi-mode interference devices (MMI devices [22]). Provided the lateral boundaries of a planar waveguide are perfectly reflecting, wave field propagation is equivalent to propagation of a periodically repeated wave field in the unbounded planar waveguide [23].

Our purpose is to find the wave field emitted by the MCF (7) after its round-trip over the AW with length $Z_T/4$ (reflectivity of mirror 2 shown in figure 1 is set equal to unity). Noting the equivalence between propagation in the AW and in the unbounded planar waveguide with periodic condition, evolution of the wave field emitted by an individual microcore can be described as propagation over fractional Talbot distance for an array with the period $2\pi R_C/N\Lambda$. The corresponding Talbot distance is $Z_{NT} = N^2Z_T$. The Green function for radiation from a point source located at $x = 0$ propagating a distance $Z_{NT}/(2N^2)$ can be derived for an even $N$ from (4):

$$u_A(Z_{TN}/2N^2, x) = \frac{1}{N\sqrt{1}} \sum_{m=-\infty}^{\infty} \exp\left(\frac{i\pi m^2}{N^2}\right) \delta(x - m\Lambda/N).$$

Thus, the single point source produces at the end of a round-trip over the AW $N^2$ replicas periodically positioned in a circle with $N$ replicas within an interval $\Lambda$ (shown as an array of images in figure 2). These replicas take positions $m\Lambda/N$ ($m = 0, 1, \ldots, N-1$) within one unit cell. For a microcore array with a sufficiently small fill factor (strictly speaking, a support of function $f(r)$ in formula (7) must fall in $1/N$th fraction of $\Lambda$), only the terms with $m = Nj$, where $j = 0, 1, 2, \ldots$, in expression (8) contribute to the field in the microcore. These terms correspond to
the images centred at the microcores positions. All the other images miss the microcore apertures spreading in the MCF and seeing no gain.

For \( N \) microcores emitting into the AW, the whole wave field at the end of the round-trip is a sum of wave field patterns produced by each of the microcores (figure 2). Summation can easily be performed using the expression (8). Thus, in the low-fill factor limit:

\[
2a/\Lambda < 1/N, \tag{9}
\]

the field distribution, which is excited in the microcores after round-trip over the AW with length \( Z_T/4 \):

\[
u_{in}(r) = \sum_{m=0}^{N-1} C_m f(r - r_m),
\]

can be calculated. It can be obtained for coefficients \( C_m \):

\[
C_m = \sum_{j=0}^{N-1} (-1)^{m+j} a_j \frac{\exp(i\varphi_j)}{N\sqrt{i}} = \frac{(-1)^m}{N\sqrt{i}} \sum_{j=0}^{N-1} (-1)^j a_j \exp(i\varphi_j). \tag{10}
\]

Thus, propagation over the distance \( Z_T/2 \) and following coupling to the microcore array results in alternating \((0, \pi)\) phase distribution with equal amplitudes for any set \((a_j, \varphi_j)\) (figure 2). This field distribution is the only mode of the MCF laser under consideration with an eigenvalue:

\[
\gamma = \sum_{j=0}^{N-1} a_j \exp(i\varphi_j)/(N\sqrt{i}). \tag{11}
\]

In particular, for identical microcores \((a_j = 1, \varphi_j = 0)\) the absolute eigenvalue \(|\gamma| = |\exp(-i\pi/4)| = 1\). It follows from equation (10) that the round trip over the AW results in optical coupling of all the microcores with the same strength. Coupling of this kind (called parallel coupling [24]) was shown in [25] to result in in-phase (or out-of-phase) array lasing even in the case of strong spread of resonator eigenfrequencies. It was shown in [26] for the linear array in a Talbot cavity, that a fill-factor smaller than \(1/N\) provides parallel coupling in the array, but with unacceptably high losses (a similar conclusion is drawn in [20]).

Figure 2. Out-of-phase mode selection in Talbot filter with length \(1/4Z_T\).
The construction analysed here displays parallel coupling between elements with no loss, if the microcores are identical, because the array elements are placed in a circle. So, as well as in MMI devices [22], there are no edge losses and no edge effects destroying the Talbot effect.

In practice, the microcore mode profile is not a function with finite support, but has wings falling away with distance from the microcore centre. This effect can be taken into account by including contributions from neighbouring images (with \( m = Nj \pm 1 \)). Performing calculations, the following expression for coefficients \( C_m \) can be derived:

\[
C_m = \frac{(-1)^m}{N \sqrt{2}i} \left\{ \sum_{j=0}^{N-1} (-1)^j a_j \exp (i\varphi_j) + S \exp \left( \frac{i\pi}{N^2} \right) \left[ \exp \left( -2i\pi m/N \right) \sum_{j=0}^{N-1} a_j \exp \left( \frac{i\varphi_j}{N} \right) \right] \right\},
\]

where an overlap integral \( S = \int f(r)f'(r - \Lambda/(2N)) \, dr \), is introduced. This expression can be interpreted as a linear combination of three super-modes: out-of-phase mode \( (K = N/2) \) and supermodes nearest to it \( (K = N/2 \pm 1, N/2 + 1) \).

When the condition (9) is fulfilled, any supermode is strictly prohibited at an odd number of microcores. The reason is that there is no overlap of the wave field incident onto the MCF with the microcores. With increasing fill factor, two supermodes nearest to the out-of-phase mode are excited. Further, the number of supermodes that can be excited grows, and the optical quality of the output combined beam worsens.

If the length of the AW is \( Z_T/2 \), the coefficients \( C_m \) can be expressed for even \( N \) as:

\[
C_m = \frac{1}{N \sqrt{2}i} \left[ (-1)^m (1 - i) \sum_{j=0}^{N-1} (-1)^j a_j \exp (i\varphi_j) + (1 + i) \sum_{j=0}^{N-1} a_j \exp (i\varphi_j) \right].
\]

Thus, \( u_{m+} \) can be presented in this case as the combination of in-phase and out-of-phase field distributions. This result is well known in the Talbot effect theory where the in-phase and out-of-phase distributions are both reproduced after propagation over the Talbot distance [20]. The weighting coefficients of the in- and out-of-phase distributions are determined by the set \( (a_j, \varphi_j) \). Provided the condition (9) holds, only the in-phase mode for the AW length \( Z_T/2 \) and an odd number of microcores is excited.

4. Phase locking of laser array with different optical lengths

Note that in the device shown in figure 1, partial phase locking was observed in the experiment [7] in spite of the fact that the microcore optical length spread was dozens of wavelengths. To explain this phenomenon, note that the phase increment due to propagation in a given microcore depends on the exact value of the radiation frequency. Residues of the phase increment division by \( 2\pi \) have uniform distribution in the interval \( 0 < \varphi < 2\pi \). These phase shifts change randomly with variation of the radiation frequency. The laser frequency adjusts...
within the spectral gain band to maximize the difference between the gain and losses. Mode attenuation is minimum when the wave field emitted by the MCF is closest to the out-of-phase one. Thus, the phase set is established, which provides the best lasing condition, while the radiation frequency is self-tuned to this condition. The idea of achieving partial phase locking at a specific frequency in a laser array with a large spread of optical lengths was first proposed in [27].

The MCF considered has length $L$ and contains $N$ microcores with radii $r_m$, which are random values with mean value $a$ and dispersion $\Delta r \ll a$. The laser radiation wavelength is $\lambda_0 + \delta \lambda$, where $\lambda_0$ is the wavelength of the spectral band centre. The phase shift in the $m$th microcore, introduced in (7), can be written as:

$$\varphi_m(\delta \lambda) \approx 4 \delta \beta L \frac{\delta r_m}{a} + 4 \delta \beta L \frac{\delta r_m \delta \lambda}{a \lambda_0},$$

(14)

where $\delta \beta \approx (2.405)^2 \lambda_0/(4\pi n a^2)$ is a term in an expression for the propagation constant responsible for transverse structure, averaged over the microcore array; $\delta r_m = r_m - a$; and $n$ is the microcore refractive index. The first term on the right-hand side of equation (14) corresponds to the phase shift of wave field with wavelength $\lambda_0$. This shift is much larger than $2\pi$ for the experimental MCF construction [7]. Thus, the phase shift $\varphi_m(0)$ is a random variable having uniform distribution in the range $[0, 2\pi]$. The last term in (14) describes the evolution of random distribution $\varphi_m$, with frequency scanned over the gain spectral band.

As shown above, the out-of-phase supermode is selected in MCF laser with the fill factor obeying (9) and AW of length equal to $\frac{1}{4} Z_T$. The absolute value of an eigenvalue for this mode $\gamma$ can be obtained from (14):

$$|\gamma(\delta \lambda)| \approx \left| N^{-1} \sum_{m=0}^{N-1} \exp \left[ 4i \delta \beta L \frac{\delta r_m}{a} \left( 1 + \frac{\delta \lambda}{\lambda_0} \right) \right] \right|. $$

(15)

The absolute value of $\gamma$ calculated by (15) at $a = 4 \mu m$, $\Delta r = 0.2 \mu m$, $L = 60 cm$, $N = 18$ the gain spectral band halfwidth $\delta \lambda_{\text{max}} = 50 \text{Å}$ is plotted in figure 3 against

Figure 3. Out-of-phase supermode eigenvalue module plotted against normalized wavelength detuning for two sets of microcore radii.
wavelength detuning, for 2 microcore radii sets $\delta r_m$, which were produced by a random number generator. It can be seen that there are values of radiation wavelength at which $|\gamma|$ is larger than 0.5.

A cavity consisting of a plane mirror, Fourier selector and an array of retro-mirrors placed at different distances from the plane mirror was studied in [27]. The asymptotic expression for axial brightness, averaged over random positions of the retro-mirror, was derived using a theory of records. This quantity is analogous in the construction under consideration to $\langle N|\gamma|^2 \rangle$, where symbols $\langle \rangle$ designate averaging over an ensemble of $\delta r_m$ sets, $|\gamma|^2$ is the maximum in the spectral gain band of the quantity $|\gamma(\delta \lambda)|^2$ calculated by the formula (15). This quantity can be approximated for $N \gg 1$ by the expression:

$$\langle N|\gamma|^2 \rangle \approx 0.577 + \ln \left[ NL \frac{\delta \lambda_{\text{max}} \Delta r}{\alpha^3 \pi} \left( \frac{2.405}{n} \right)^2 \right].$$  \hspace{1cm} (16)$$

Values of $\langle |\gamma|^2 \rangle$ are plotted against MCF length in figure 4, both calculated by formula (16), and averaged over 50 MCF realizations for the parameters values in figure 3. It can be seen that the formula (16) gives a rather good approximation of numerical results. Phase locking efficiency looks to be saturated when MCF length is 1–2 m.

The fill factor in the experiment [7] did not satisfy the condition (9), so the theory described above gives a general idea about modal properties of this cavity. The condition for expanding the beam emitted by the microcore to the entire array due to diffraction is also relation (9). Thus, several round-trips through the cavity are necessary to couple all the microcores. The result is that the oscillating transverse mode intensity is distributed non-uniformly over the array, making easier competition for other modes. Nevertheless, numerical simulations made in [28] show that it is possible to use a low fill-factor model even in this case. Substantial improvement in output beam quality and reduced losses were found at the frequency corresponding to the maximum evaluated by formula (15).
5. Conclusions
Phase-locking of a multi-core fibre laser incorporating an annular waveguide has been considered. It has been shown that parallel coupling of microcores is achieved for sufficiently small fill factors due to a fractional Talbot effect. The out-of-phase supermode is selected if the AW length is one-quarter of the Talbot distance. It has been shown that phase-locking can occur for a large spread of microcore optical path lengths due to the lasing frequency self-tuning in the spectral gain band. Parallel coupling in the identical emitter array can be realized without losses, in contrast to experiments with linear laser array phase-locking [29, 30]. The difference is that there are no edge effects due to the ring arrangement of microcores.

Notice that there is a fundamental limitation to laser array coupling in the Talbot cavity discussed above due to breaking of the paraxial approximation. The aberrations destroy self-reproduction (see [18]) diminishing the coupling between the array elements. It was shown in [31] that the condition for small aberrations over the array results in a restriction on the fill factor: \(2a/\Lambda > \lambda/(\pi^2a)\). This condition in combination with limitation (9) gives the maximum number of microcores, which can be effectively coupled:

\[
N \leq \frac{\pi^2 \cdot 2a}{\lambda}.
\] (17)

This number is about 40 for the construction considered and can be increased by taking larger \(a\) (it necessary to reduce the microcore numerical aperture).

Parallel coupling was claimed in [32] to occur in 1-to-\(N\)-way cavity based on a planar waveguide, but, to our knowledge, no experimental evidence was presented. The authors [32] proposed to exploit the image multiplication effect at \(1/N\)th of the Talbot distance to couple \(N\) elements to one. The condition (9) is satisfied in this construction automatically, but the restriction (17) is important, putting a limit on the maximum number of elements that can be coupled.

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